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$U(1)_Q$ invariance and $SU(3)_C \otimes SU(3)_\mathrm{L} \otimes U(1)_X$ models with β arbitrary

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Abstract. Using $U(1)_Q$ invariance, the photon eigenstate and the matching gauge coupling constants in $SU(3)_C \otimes SU(3)_\mathrm{L} \otimes U(1)_X$ models with arbitrary β are given. The mass matrix of the neutral gauge bosons is exactly diagonalized, and the photon eigenstate is independent on the symmetry breaking parameters – the VEVs of the Higgs scalars. By obtaining the electromagnetic vertex, the model is embedded naturally into the standard model.

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1 Introduction

The detection of neutrino oscillations [1] experimentally indicates that neutrinos are massive particles and that flavor lepton number is not conserved. Since in the standard model (SM), neutrinos are massless and flavor lepton number is conserved, neutrino oscillation experiments are a clear sign that the SM has to be extended.

A very common alternative to solve some of the problems of the SM consists of enlarging the group of the gauge symmetry, where the larger group properly embeds the SM. For instance, the $SU(5)$ grand unification model [2] can unify the interactions and predicts electric charge quantization, while the group E_6 can also unify the interactions and might explain the masses of the neutrinos [3], etc. [4]. Nevertheless, such models cannot explain the generation number problem.

A very interesting alternative to explain the origin of generations comes from the cancelation of chiral anomalies [5]. In particular, models with the gauge group $G_{331} = SU(3)_C \otimes SU(3)_\mathrm{L} \otimes U(1)_X$, also called 3-3-1 models [6–8], arise as a possible solution to this puzzle, since some of such models require three generations in order to cancel chiral anomalies completely. An additional motivation to study this kind of models comes from the fact that they can also predict charge quantization [9].

In the literature on 3-3-1 models, it is known that the matching of gauge coupling constants at the $SU(3)_\mathrm{L} \otimes U(1)_X$ breaking is dependent on the constraints among the VEVs [6]. In addition, the independence on the VEVs of the photon eigenstate and mass has not been explained yet [10].

In this paper, we have pointed out that the photon eigenstate is independent on the VEVs; and the matching of gauge coupling constants is not dependent on VEVs structure.

This paper is organized as follows. In Sect. 2 we recall some features of the 3-3-1 models with arbitrary β and study the mass Lagrangian of the neutral gauge bosons, the photon eigenstate and mass. Matching gauge coupling constants and diagonalizing the neutral bosons gauge mass matrix are obtained in Sect. 3. Finally, our conclusions are summarized in the last section.

2 Photon eigenstate

The 3-3-1 model with arbitrary β [11] has the electric charge operator in the following form:

$$Q = T_3 + \beta T_8 + X, \tag{2.1}$$

where $T_3 = \lambda_3/2, T_8 = \lambda_8/2$ are the $SU(3)_\mathrm{L}$ gauge charges, and X is the $U(1)_X$ gauge charge.

Under the gauge symmetry G_{331}, the fermion representations are given by the triplets $\mathbf{3}$, antitriplets $\mathbf{3}^*$, and singlets $\mathbf{1}$ (for the right-handed counterparts) of the $SU(3)_\mathrm{L}$ group. In order to cancel the anomalies, the same number of fermion triplets and antitriplets must be present [11].

A triplet of the $SU(3)_\mathrm{L}$ group is composed of a doublet $\mathbf{2}$ and a singlet $\mathbf{1}$ of the $SU(2)_\mathrm{L}$ group of the SM; therefore, it is decomposed as follows:

$$\Big(u,\, d,\, s\Big)^\mathrm{T} = \Big(u,\, d\Big)^\mathrm{T} \oplus s, \tag{2.2}$$

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or

$$(3, X) = (2, X) \oplus (1, X), \quad (2.3)$$

where u, d, and s denote the first and the second members of the doublets, and of the singlets, respectively. In the case of an antitriplet $\mathbf{3}^*$, it is also decomposed into an antidoublet $\mathbf{2}^*$ and a singlet of the $SU(2)_{\mathrm{L}}$ group:

$$\left(d,\, -u,\, s'\right)^{\mathrm{T}} = \left(d,\, -u\right)^{\mathrm{T}} \oplus s', \quad (2.4)$$

or

$$(3^*, -X) = (2^*, -X) \oplus (1, -X). \quad (2.5)$$

To find the hyper-charge Y of the doublets and the singlets, we should use $Y = 2(\beta T_8 + X)$ which is obtained directly from (2.1).

Spontaneous symmetry breaking from the G_{331} to the G_{SM} group of the SM [12] will allow the singlet member to be separated from the triplets or antitriplets and get mass. This is achieved by a Higgs scalar triplet χ with the VEV as follows:

$$\langle\chi\rangle^{\mathrm{T}} = \left(0, 0, \frac{v_s}{\sqrt{2}}\right). \quad (2.6)$$

Then the neutral gauge bosons of the theory get mass from

$$\mathcal{L}^{\chi}_{\mathrm{mass}} = (D^H_\mu \langle\chi\rangle)^+ (D^{H\mu}\langle\chi\rangle), \quad (2.7)$$

where the subscripts H denote the diagonal part of the covariant derivative

$$D^H_\mu = \partial_\mu + \mathrm{i}gT_3W^3_\mu + \mathrm{i}gT_8W^8_\mu + \mathrm{i}g_X T_9 X_\chi B_\mu. \quad (2.8)$$

Here g and g_X are the gauge coupling constants of the $SU(3)_{\mathrm{L}}$ and $U(1)_X$ groups, respectively. X_χ is the $U(1)_X$ charge of the χ Higgs scalar. $T_9 = \mathrm{diag}(1,1,1)/\sqrt{6}$ is chosen such that $\mathrm{Tr}(T_aT_b) = \delta_{ab}/2$; $a, b = 1, 2, ..., 9$. Substituting the charge X_χ from (2.1) into (2.8), we get

$$\begin{aligned} D^H_\mu &= \partial_\mu + \mathrm{i}gT_3W^3_\mu + \mathrm{i}gT_8W^8_\mu \\ &\quad + \mathrm{i}\frac{g_X}{\sqrt{6}} B_\mu \left(Q - T_3 - \beta T_8\right). \end{aligned} \quad (2.9)$$

The $U(1)_Q$ invariance requires $Q\langle\chi\rangle = 0$; therefore, we get

$$\begin{aligned} D^H_\mu \langle\chi\rangle &= \frac{\mathrm{i}gv_s}{2\sqrt{2}} \left(-\frac{2}{\sqrt{3}}W^8_\mu + \frac{2t}{\sqrt{6}}B_\mu \frac{\beta}{\sqrt{3}}\right) \\ &= \frac{\mathrm{i}gv_s}{2\sqrt{2}} \varDelta_{3\mu}, \end{aligned} \quad (2.10)$$

where the following notation is used:

$$\varDelta_{3\mu} \equiv \left(-\frac{2}{\sqrt{3}}W^8_\mu + \frac{2t}{\sqrt{6}}B_\mu \frac{\beta}{\sqrt{3}}\right), \quad (2.11)$$

$$t \equiv g_X/g. \quad (2.12)$$

Hence

$$\mathcal{L}^{\chi}_{\mathrm{mass}} = \frac{g^2 {v_s}^2}{8} \varDelta^2_3, \quad (2.13)$$

where $\varDelta^2_3 = \varDelta_{3\mu}\varDelta^\mu_3$.

In the second step of symmetry breaking [5,6,12], the G_{SM} group must be decomposed into the $SU(3)_C \otimes U(1)_Q$ group, and two η, ρ Higgs triplets are introduced with the following VEVs:

$$\begin{aligned} \langle\eta\rangle^{\mathrm{T}} &= \left(\frac{v_u}{\sqrt{2}}, 0, 0\right), \\ \langle\rho\rangle^{\mathrm{T}} &= \left(0, \frac{v_d}{\sqrt{2}}, 0\right). \end{aligned} \quad (2.14)$$

The neutral gauge bosons also gain mass from two Lagrangians given by

$$\mathcal{L}^{\eta}_{\mathrm{mass}} = (D^H_\mu \langle\eta\rangle)^+ (D^{H\mu}\langle\eta\rangle), \quad (2.15)$$

$$\mathcal{L}^{\rho}_{\mathrm{mass}} = (D^H_\mu \langle\rho\rangle)^+ (D^{H\mu}\langle\rho\rangle). \quad (2.16)$$

Noting that $Q\langle\eta\rangle = Q\langle\rho\rangle = 0$, we get

$$\begin{aligned} D^H_\mu \langle\eta\rangle &= \frac{\mathrm{i}gv_u}{2\sqrt{2}} \left[W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu + \frac{2t}{\sqrt{6}}B_\mu \left(-\frac{1}{2} - \frac{\beta}{2\sqrt{3}}\right)\right] \\ &= \frac{\mathrm{i}gv_u}{2\sqrt{2}} \varDelta_{1\mu}, \end{aligned} \quad (2.17)$$

$$\begin{aligned} D^H_\mu \langle\rho\rangle &= \frac{\mathrm{i}gv_d}{2\sqrt{2}} \left[-W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu + \frac{2t}{\sqrt{6}}B_\mu \left(\frac{1}{2} - \frac{\beta}{2\sqrt{3}}\right)\right] \\ &= \frac{\mathrm{i}gv_d}{2\sqrt{2}} \varDelta_{2\mu}; \end{aligned} \quad (2.18)$$

here

$$\varDelta_{1\mu} \equiv \left[W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu + \frac{2t}{\sqrt{6}}B_\mu \left(-\frac{1}{2} - \frac{\beta}{2\sqrt{3}}\right)\right], \quad (2.19)$$

$$\varDelta_{2\mu} \equiv \left[-W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu + \frac{2t}{\sqrt{6}}B_\mu \left(\frac{1}{2} - \frac{\beta}{2\sqrt{3}}\right)\right]. \quad (2.20)$$

Hence

$$\begin{aligned} \mathcal{L}^{\eta}_{\mathrm{mass}} &= \frac{g^2 v_u^2}{8} \varDelta^2_1, \\ \mathcal{L}^{\rho}_{\mathrm{mass}} &= \frac{g^2 v_d^2}{8} \varDelta^2_2. \end{aligned} \quad (2.21)$$

Finally, the mass Lagrangian of the neutral gauge bosons is given by

$$\begin{aligned} \mathcal{L}^{\mathrm{NCC}}_{\mathrm{mass}} &= \mathcal{L}^{\eta}_{\mathrm{mass}} + \mathcal{L}^{\rho}_{\mathrm{mass}} + \mathcal{L}^{\chi}_{\mathrm{mass}}, \\ &= \frac{g^2}{8} \left(v_u^2 \varDelta^2_1 + v_d^2 \varDelta^2_2 + v_s^2 \varDelta^2_3\right). \end{aligned} \quad (2.22)$$

In general, any 3-3-1 model needs to have three Higgs triplets for breaking the G_{331} group into the $SU(3)_C \otimes U(1)_Q$ group. However, some 3-3-1 models need less than three Higgs triplets [13]. For our purpose in obtaining the mass matrix of the neutral gauge bosons, this corresponds to vanishing v_u or v_d. In the case with more than three Higgs triplets, one just makes the following appropriate replacements:

$$v_u^2 \to v_u^2 + v_{u1}^2 + v_{u2}^2 + ...,$$

$$v_d^2 \to v_d^2 + v_{d1}^2 + v_{d2}^2 + ...,$$
$$v_s^2 \to v_s^2 + v_{s1}^2 + v_{s2}^2 + ..., \tag{2.23}$$

where v_{ui}, v_{dj}, v_{sk} are the VEVs of the additional Higgs triplets. They belong to the up, down, and singlet members, respectively. This also remains correct for the cases if a Higgs triplet has two neutral members with the non-zero VEVs [13], and for models with Higgs antitriplets.

In some models, for example the minimal 3-3-1 model [6], to give mass to all leptons, we have to introduce a Higgs sextet. Let us denote the Higgs sextet by Γ_{ij}. Then, the mass Lagrangian will get an addition

$$\mathcal{L}^{\Gamma}_{\text{mass}} = (D^H_\mu \langle\Gamma\rangle_{ij})^+ (D^{H\mu}\langle\Gamma\rangle_{ij}), \tag{2.24}$$

where

$$\begin{aligned} D^H_\mu\langle\Gamma\rangle_{ij} &= \mathrm{i}g\left[(W^3_\mu T_3 + W^8_\mu T_8)^k_i \langle\Gamma\rangle_{kj} \right. \\ &\quad \left. + (W^3_\mu T_3 + W^8_\mu T_8)^k_j \langle\Gamma\rangle_{ki}\right] + \frac{\mathrm{i}g_X}{\sqrt{6}} X_\Gamma B_\mu \langle\Gamma\rangle_{ij} \\ &= \mathrm{i}g\left[\left(W^3_\mu T_3 + W^8_\mu T_8 + \frac{t}{\sqrt{6}}(Q - T_3 - \beta T_8)B_\mu\right)^k_i \langle\Gamma\rangle_{kj}\right] \\ &\quad + \mathrm{i}g\left[\left(W^3_\mu T_3 + W^8_\mu T_8 + \frac{t}{\sqrt{6}}(Q - T_3 - \beta T_8)B_\mu\right)^k_j \langle\Gamma\rangle_{ki}\right] \\ &= \frac{\mathrm{i}g}{2}\begin{pmatrix} 2\langle\Gamma\rangle_{11}\Delta_{1\mu} & \langle\Gamma\rangle_{12}(\Delta_{1\mu}+\Delta_{2\mu}) & \langle\Gamma\rangle_{13}(\Delta_{1\mu}+\Delta_{3\mu}) \\ \langle\Gamma\rangle_{12}(\Delta_{1\mu}+\Delta_{2\mu}) & 2\langle\Gamma\rangle_{22}\Delta_{2\mu} & \langle\Gamma\rangle_{23}(\Delta_{2\mu}+\Delta_{3\mu}) \\ \langle\Gamma\rangle_{13}(\Delta_{1\mu}+\Delta_{3\mu}) & \langle\Gamma\rangle_{23}(\Delta_{2\mu}+\Delta_{3\mu}) & 2\langle\Gamma\rangle_{33}\Delta_{3\mu} \end{pmatrix}. \end{aligned} \tag{2.25}$$

It is easy to verify that

$$\Delta_{1\mu} + \Delta_{2\mu} + \Delta_{3\mu} = 0. \tag{2.26}$$

Finally, the mass term for neutral gauge bosons from the sextet is given by

$$\begin{aligned} \mathcal{L}^{\Gamma}_{\text{mass}} = \frac{g^2}{2}&\left\{[2\langle\Gamma\rangle^2_{11} + \langle\Gamma\rangle^2_{23}]\Delta_1^2 + [2\langle\Gamma\rangle^2_{22} + \langle\Gamma\rangle^2_{13}]\Delta_2^2 \right. \\ &\left. + [2\langle\Gamma\rangle^2_{33} + \langle\Gamma\rangle^2_{12}]\Delta_3^2\right\}. \end{aligned} \tag{2.27}$$

Note that only neutral members in the sextet can have non-zero VEVs. From (2.27) we see that the general form of the mass Lagrangian (2.22) is not changed by adding $\mathcal{L}^{\Gamma}_{\text{mass}}$.

In order to generate the fermion masses, the Higgs bosons should lie in either the triplet, antitriplet, sextet, or singlet representation of $SU(3)_{\mathrm{L}}$ [12]. In the latter case, the singlet must be neutral, and it does not give mass to gauge bosons. So, we conclude that for any 3-3-1 model, the mass matrix of the neutral gauge bosons always has the form given in (2.22).

The mass Lagrangian (2.22) can be rewritten

$$\mathcal{L}_{\text{mass}} = \frac{1}{2}V^{\mathrm{T}} M^2 V, \tag{2.28}$$

where $V^{\mathrm{T}} = (W^3, W^8, B)$, and

$$M^2 = \frac{1}{4}g^2\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33}\end{pmatrix}, \tag{2.29}$$

with

$$\begin{aligned} m_{11} &= v_u^2 + v_d^2, \\ m_{12} &= \frac{1}{\sqrt{3}}\left(v_u^2 - v_d^2\right), \\ m_{13} &= \frac{t}{\sqrt{6}}\left[v_u^2\left(-1 - \frac{\beta}{\sqrt{3}}\right) - v_d^2\left(1 - \frac{\beta}{\sqrt{3}}\right)\right], \\ m_{22} &= \frac{1}{3}\left(v_u^2 + v_d^2 + 4v_s^2\right), \\ m_{23} &= \frac{t}{3\sqrt{2}}\left[v_u^2\left(-1 - \frac{\beta}{\sqrt{3}}\right) + v_d^2\left(1 - \frac{\beta}{\sqrt{3}}\right) - v_s^2\frac{4\beta}{\sqrt{3}}\right], \\ m_{33} &= \frac{t^2}{6}\left[v_u^2\left(-1 - \frac{\beta}{\sqrt{3}}\right)^2 + v_d^2\left(1 - \frac{\beta}{\sqrt{3}}\right)^2 \right. \\ &\quad \left. + v_s^2\left(\frac{2\beta}{\sqrt{3}}\right)^2\right]. \end{aligned} \tag{2.30}$$

It can be checked that the matrix M^2 has a *non-degenerate* zero eigenvalue for any breaking parameters in any 3-3-1 model. Therefore, the zero eigenvalue is identified with the photon mass, $M_\gamma^2 = 0$.

The eigenstate with the zero eigenvalue can be obtained directly from the following equation:

$$M^2\begin{pmatrix} A_{\gamma 1} \\ A_{\gamma 2} \\ A_{\gamma 3}\end{pmatrix} = 0. \tag{2.31}$$

We then get

$$A_\gamma = \begin{pmatrix} t \\ \beta t \\ \sqrt{6}\end{pmatrix}\frac{1}{\sqrt{6 + (1+\beta^2)t^2}}. \tag{2.32}$$

So the physical photon field A_μ is given by

$$\begin{aligned} A_\mu = &\frac{t}{\sqrt{6+(1+\beta^2)t^2}}W^3_\mu + \frac{\beta t}{\sqrt{6+(1+\beta^2)t^2}}W^8_\mu \\ &+ \frac{\sqrt{6}}{\sqrt{6+(1+\beta^2)t^2}}B_\mu. \end{aligned} \tag{2.33}$$

For any 3-3-1 model, we see that the photon eigenstate and mass are not dependent on the VEVs (v_u, v_d, v_s). These are a natural consequence of the $U(1)_Q$ invariance – the conservation of the electric charge.

3 Matching gauge coupling constants

To embed the 3-3-1 model into the SM, we will work with the electromagnetic vertex. Let us denote two remain massive fields by Z^1_μ, Z^2_μ. We change basis by the unitary matrix

$$(A_\mu, Z^1_\mu, Z^2_\mu) = (W^3_\mu, W^8_\mu, B_\mu)U, \tag{3.1}$$

where U has the following form:

$$U = \begin{pmatrix} \frac{t}{\sqrt{6+(1+\beta^2)t^2}} & U_{12} & U_{13} \\ \frac{\beta t}{\sqrt{6+(1+\beta^2)t^2}} & U_{22} & U_{23} \\ \frac{\sqrt{6}}{\sqrt{6+(1+\beta^2)t^2}} & U_{32} & U_{33} \end{pmatrix}. \quad (3.2)$$

Here the elements in the second and third columns need not necessarily be determined. From (3.1) and (3.2), we get

$$W^3_\mu = \frac{t}{\sqrt{6+(1+\beta^2)t^2}} A_\mu + U_{12} Z^1_\mu + U_{13} Z^2_\mu,$$
$$W^8_\mu = \frac{\beta t}{\sqrt{6+(1+\beta^2)t^2}} A_\mu + U_{22} Z^1_\mu + U_{23} Z^2_\mu,$$
$$B_\mu = \frac{\sqrt{6}}{\sqrt{6+(1+\beta^2)t^2}} A_\mu + U_{32} Z^1_\mu + U_{33} Z^2_\mu. \quad (3.3)$$

The interactions among the gauge bosons and fermions are given by

$$\mathcal{L}_F = \bar{R} \mathrm{i}\gamma^\mu \left(\partial_\mu + \mathrm{i}\frac{g_X}{\sqrt{6}} X B_\mu\right) R + \bar{L} \mathrm{i}\gamma^\mu \left(\partial_\mu + \mathrm{i} g W^a_\mu \frac{\lambda_a}{2} + \mathrm{i}\frac{g_X}{\sqrt{6}} X B_\mu\right) L, \quad (3.4)$$

where R represents any right-handed singlet and L any left-handed triplet or antitriplet. Substituting W^3, W^8, B from (3.3) into (3.4), for $R = e_\mathrm{R}$ with $X_{e_\mathrm{R}} = -1$ and $L = (\nu_{e\mathrm{L}}, e_\mathrm{L}, E_\mathrm{L})^\mathrm{T}$ with $X_\mathrm{L} = -1/2 - \beta/2\sqrt{3}$, we get

$$\begin{aligned} \mathcal{L}^{\mathrm{int}}_{\bar{e}e\gamma} &= -\bar{e}_\mathrm{R} \mathrm{i}\gamma^\mu \left[\frac{\mathrm{i} g_X}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6+(1+\beta^2)t^2}}\right] A_\mu e_\mathrm{R} \\ &+ \bar{e}_\mathrm{L} \mathrm{i}\gamma^\mu \left[-\frac{\mathrm{i}g}{2}\frac{t}{\sqrt{6+(1+\beta^2)t^2}} \right. \\ &\quad + \frac{\mathrm{i}g}{2\sqrt{3}} \frac{\beta t}{\sqrt{6+(1+\beta^2)t^2}} \\ &\quad \left. - \frac{\mathrm{i} g_X}{\sqrt{6}} \left(\frac{1}{2} + \frac{\beta}{2\sqrt{3}}\right) \frac{\sqrt{6}}{\sqrt{6+(1+\beta^2)t^2}}\right] A_\mu e_\mathrm{L} \\ &= \frac{g_X}{\sqrt{6+(1+\beta^2)t^2}} \bar{e}\gamma^\mu e A_\mu. \end{aligned} \quad (3.5)$$

The coefficient of the $\bar{e}e\gamma$ vertex in (3.5) is identified with the electromagnetic coupling constant

$$\frac{g_X}{\sqrt{6+(1+\beta^2)t^2}} \equiv e. \quad (3.6)$$

In the SM, we have

$$\frac{g_2 g_Y}{\sqrt{g_2^2 + g_Y^2}} = e, \quad (3.7)$$

where g_2, g_Y are coupling constants of the $SU(2)_\mathrm{L}$ and $U(1)_Y$ gauge group, respectively. Using continuation of the gauge coupling constant of the $SU(3)_\mathrm{L}$ group at the spontaneous symmetry breaking point,

$$g = g_2 \equiv g, \quad (3.8)$$

from (3.6) and (3.7), we get

$$\frac{1}{g_Y^2} = \frac{\beta^2}{g^2} + \frac{6}{g_X^2}. \quad (3.9)$$

From (3.6), we obtain

$$\frac{t}{\sqrt{6+(1+\beta^2)t^2}} = \frac{e}{g}. \quad (3.10)$$

As in the SM, we put

$$\frac{t}{\sqrt{6+(1+\beta^2)t^2}} = \sin\theta_\mathrm{W}. \quad (3.11)$$

Equation (3.9) yields

$$t = \frac{\sqrt{6} t_\mathrm{W}}{\sqrt{1-\beta^2 t_\mathrm{W}^2}}. \quad (3.12)$$

Hence, the photon eigenstate can be rewritten in the form

$$A_\mu = s_\mathrm{W} W^3_\mu + c_\mathrm{W}\left(\beta t_\mathrm{W} W^8_\mu + \sqrt{1-\beta^2 t_\mathrm{W}^2} B_\mu\right). \quad (3.13)$$

From the orthogonal condition of the photon eigenstate to the two remaining gauge vectors, we can write

$$Z_\mu = c_\mathrm{W} W^3_\mu - s_\mathrm{W}\left(\beta t_\mathrm{W} W^8_\mu + \sqrt{1-\beta^2 t_\mathrm{W}^2} B_\mu\right), \quad (3.14)$$

$$Z'_\mu = \sqrt{1-\beta^2 t_\mathrm{W}^2} W^8_\mu - \beta t_\mathrm{W} B_\mu. \quad (3.15)$$

Therefore, in this basis, the mass matrix $M^2 \to M^{2'}$ has the following form:

$$M^{2'} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_Z^2 & M_{ZZ'}^2 \\ 0 & M_{ZZ'}^2 & M_{Z'}^2 \end{pmatrix}, \quad (3.16)$$

where

$$\begin{aligned} M_Z^2 &= \frac{g^2}{4c_\mathrm{W}^2}(v_u^2 + v_d^2), \\ M_{ZZ'}^2 &= \frac{g^2}{4\sqrt{3}c_\mathrm{W}^2\sqrt{1-(1+\beta^2)s_\mathrm{W}^2}} \\ &\times \left\{\left[\left(\sqrt{3}\beta - 1\right) s_\mathrm{W}^2 + 1\right] v_u^2 + \left[\left(\sqrt{3}\beta + 1\right) s_\mathrm{W}^2 - 1\right] v_d^2\right\}, \\ M_{Z'}^2 &= \frac{g^2}{12\,(1-\beta^2 t_\mathrm{W}^2)} \\ &\times \left[\left(1+\sqrt{3}\beta t_\mathrm{W}^2\right)^2 v_u^2 + \left(1-\sqrt{3}\beta t_\mathrm{W}^2\right)^2 v_d^2 + 4v_s^2\right]. \end{aligned} \quad (3.17)$$

The matrix $M^{2'}$ gives the mixings between Z_μ and Z'_μ; by rotating over an angle ϕ in the plane $(Z_\mu, Z'_\mu) \to (Z^1_\mu, Z^2_\mu)$, the mass eigenvectors are

$$\begin{aligned} Z^1_\mu &= Z_\mu \cos\phi - Z'_\mu \sin\phi, \\ Z^2_\mu &= Z_\mu \sin\phi + Z'_\mu \cos\phi, \end{aligned} \tag{3.18}$$

where ϕ is defined by

$$\tan^2\phi = \frac{M^2_Z - M^2_{Z^1}}{M^2_{Z^2} - M^2_Z}, \tag{3.19}$$

and the physical mass eigenvalues

$$M^2_{Z^1} = \frac{1}{2}\left[M^2_Z + M^2_{Z'} - \sqrt{\left(M^2_Z - M^2_{Z'}\right)^2 + 4\left(M^2_{ZZ'}\right)^2}\right], \tag{3.20}$$

$$M^2_{Z^2} = \frac{1}{2}\left[M^2_Z + M^2_{Z'} + \sqrt{\left(M^2_Z - M^2_{Z'}\right)^2 + 4\left(M^2_{ZZ'}\right)^2}\right]. \tag{3.21}$$

From the mixing mass matrix of Z and Z' we see that $\phi = 0$ if $v_s \gg v_u, v_d$ or

$$v^2_u = \frac{1 - \left(\sqrt{3}\beta + 1\right) s^2_{\mathrm{W}}}{1 + \left(\sqrt{3}\beta - 1\right) s^2_{\mathrm{W}}} v^2_d. \tag{3.22}$$

Here A, Z^1 correspond to the neutral gauge bosons of the SM (γ, Z), and Z^2 is a new neutral gauge boson.

To finish this section, we note that the matching condition of the coupling constants (3.6) at the $SU(3)_{\mathrm{L}} \otimes U(1)_X$ breaking is very obvious as the matching in the SM. It is not dependent on the constraint $v_s \gg v_u, v_d$ as in the literature [6]. After the matching, we rewrote the photon field with the coefficients in the Weinberg mixing angle, and then taking the exact diagonalization of the mass matrix for the neutral gauge bosons.

4 Conclusion

In this paper, the photon eigenstate and the matching of the coupling constants in 3-3-1 models are obtained in general form containing Higgs triplets, antitriplets as well as the sextet. We emphasized that the matching of the coupling constants is not dependent on the condition that the vacuum expectation value of the Higgs boson of the first step of breaking symmetry must be much larger than those of the second step, namely $\langle s \rangle \gg \langle u \rangle, \langle d \rangle$.

This technique can be extended for electroweak models which are based on the larger gauge groups such as $SU(3)_C \otimes SU(4)_{\mathrm{L}} \otimes U(1)_X$ [14].

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